Determining Fiber Optic Cable Numerical Aperture

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1 Purpose

Find the numerical aperture of an optical fiber through experiment and determine if it falls within experimental error of the manufacturer's listed value.

2 Introduction

Numerical aperture is a number representing the sine of the acceptance angle.

$$NA = \sin\theta_{acceptance} \tag{2.1}$$

The numerical aperture can also be related to the refractive index of the core and cladding of an optical fiber.

$$NA = \sqrt{n_{core}^2 + n_{cladding}^2} \tag{2.2}$$

Since the refraction entering and exiting the fiber is the same, and the reflection inside the fiber will preserve its angles of incidence and reflection, the entrance and departure angles will be the same. From this, the acceptance angle can be calculated from the exit angle. Using the radius and length of a beam leaving the cable, the angle can be calculated.

$$\theta = \tan^{-1}\frac{r}{l} \tag{2.3}$$

Radius is calculated from the diameter of a beam.

$$r = \frac{1}{2}d \tag{2.4}$$

3 Equipment and Procedure

Equipment Required:

- Class 1, low power, laser transmission source
- Opaque, to the laser source, surface
- Linear measurement tool
- Multiple samples of fiber optic cable in question

Equipment Used:

- Laser of JDS PS3 Series PDL Multimeter
- Generic lab laser shield
- Vernier calipers
- 5 samples of Super Eska^{TM} Simplex Optical Fiber Cable
  - $10[cm]$
  - $Medium_1$
  - $Long_1$
  - $Medium_2$
  - $Long_2$

Procedure:

1. Connect an optical fiber cable in question to the transmission source
2. Setup the opaque surface at an accessible distance opposite of the optical cable
3. Direct the open, not connected, end of the cable towards the target
4. Measure the distance from the open end of the cable to the target as l
5. Measure the diameter of the beam exiting the fiber that appears against the target as d
6. Repeat all steps for each cable sample

4 Observations and Calculations

Fibers	$l[cm]$	$\Delta l[cm]$	$d[cm]$	$\Delta d[cm]$
$10[cm]$	0.9	0.15	1.4	0.1
$Medium_1$	0.8	0.15	1.0	0.1
$Long_1$	1.3	0.15	1.0	0.1
$Long_2$	1.5	0.15	1.0	0.1
$Medium_2$	1.3	0.15	1.0	0.1

Table 4.1: Cable data and observations.

Values from cable data sheet:
$NA_{accepted} = 0.5$
$n_{core} = 1.49$

Measurement uncertainties $[cm]$:
$percision = 0.05$
$laser\ blur = 0.02$
$measure\ error = 0.03$
$fiber\ positioning = 0.05$

Sample calculation of the radius using (2.4) for the $10[cm]$ fiber:

$$\Delta r_1 = a \times d\left(\frac{\Delta a}{a} + \frac{\Delta d}{d}\right) = \frac{1}{2} \times 1.4\left(\frac{0}{0.5} + \frac{0.1}{1.4}\right) = 0.05$$

$$r_1 = d \times \frac{1}{2} = 1.4 \times \frac{1}{2} = 0.7$$

$$\Rightarrow 0.7 \pm 0.05$$

Sample calculation of the acceptance angle using (2.3) for the $10[cm]$ fiber:

$$\Delta\frac{r}{l} = \frac{r}{l}\left(\frac{\Delta r}{r} + \frac{\Delta l}{l}\right) = \frac{0.7}{0.9}\left(\frac{0.02}{0.7} + \frac{0.04}{0.15}\right) = 0.15$$

$$\Delta\theta_1 = \frac{1}{\left(\frac{r}{l}\right)^2 + 1} \times \Delta\frac{r}{l} = \tan^{-1}\left(\frac{0.7}{0.9}\right) = 38$$

$$\Rightarrow 38 \pm 0.12$$

Sample calculation of the numerical aperture using (2.1) for the 10[cm] fiber:

$$\Delta NA_1 = \Delta\theta_1 |\cos\theta_1| = 0.12|\cos 38| = 0.10$$

$$NA_1 = \sin\theta_1 = sin38 = 0.62$$

$$\Rightarrow 0.62 \pm 0.7$$

Fibers	r	Δr	θ	$\Delta\theta$	NA	ΔNA
10[cm]	0.7	0.05	38	0.12	0.62	0.10
$Medium_1$	0.5	0.05	32	0.13	0.53	0.11
$Long_1$	0.5	0.05	21	0.07	0.36	0.06
$Long_2$	0.5	0.05	18	0.06	0.31	0.06
$Medium_1$	0.5	0.05	21	0.07	0.36	0.07

Table 4.2: Summary of calculated values for all observations.

Finding the average numerical aperture:

$$\overline{NA} = \frac{0.62 + 0.53 + 0.36 + 0.31 + 0.36}{5} = 0.44$$

Uncertainty can be found by using the most significant value of the standard deviation of the mean or the precision measure

$$\alpha = \frac{1}{\sqrt{5}} \sqrt{\sum_{i=1}^{5} \frac{\left(NA_i - \overline{NA}\right)^2}{5}} = 0.06$$

$$\Delta\overline{NA} = \max\left(\alpha, percision\right) = \max\left(0.06, 0.05\right) = 0.06$$

resulting in the experimental value for the numerical aperture

$$NA = 0.04 \pm 0.06.$$

Using the calculated average numerical aperture and the known value of the core refractive index, the cladding refractive index can be found by rearranging (2.2)

$$\Rightarrow n_{cladding} = \sqrt{n_{core}^2 - NA^2} = \sqrt{1.49^2 - 0.44^2}$$

$$\Delta n_{cladding} = \frac{NA \times \Delta NA}{\sqrt{n_{core}^2 - NA^2}} = \frac{0.44 \times 0.06}{\sqrt{1.49^2 - 0.44^2}} = 0.22$$

$$\Rightarrow n_{cladding} = 1.42 \pm 0.02.$$

From rearranging (2.1) the acceptance angle of the average numerical aperture is

$$\Rightarrow \theta_{accepted} = \sin^{-1} NA = sin^{-1} 0.44 = 26.$$

An agreement analysis shows

$$|NA_{accepted} - NA| \leq \Delta NA_{accepted} + \Delta NA$$

$$\Rightarrow |0.5 - 0.44| \leq 0 + 0.06$$

$$\Rightarrow 0.06 \leq 0.06$$

which determines that the experimental value agrees with the accepted value within experimental error.

5 Discussion

The fibers were expected to have the same results since they are all from the same manufacturer. The results trend towards a smaller numerical aperture as the experiment was conducted, but the averaging of the results created an agreeable numerical aperture so the variation must not have been too significant.

Sources of error include the precision measure of the ruler used of $0.05[cm]$, the fading of the laser diameter along the edge of $0.02[cm]$. Random error created from executing the experiment include $0.05[cm]$ from positioning the fiber to measure the diameter of the laser, and $0.03[cm]$ from measuring on angles. The fiber positioning occurred only in the length measurement.

6 Conclusion

The numerical aperture was calculated to be 0.44 ± 0.6 which agrees to the accepted value within experimental uncertainty.